Exercises

13.3.1 Evaluate each of the following without a calculator:

(a) $\log_3\left(27^{2007}\right)$

(b) $6^{\log_6 418}$

(c) $\log_2 \frac{2}{3} + \log_2 6$

(d) $(\log_2 5)(\log_5 12) + (\log_2 7)(\log_7 \frac{8}{3})$

(e) $\dfrac{\log_2 125}{\log_2 25}$

(f)★ $\dfrac{2^{\log_4 108}}{2^{\log_4 3}}$

13.3.2 Prove that $\log_b \frac{1}{a} = -\log_b a$.

13.3.3 If $c = \log_y b$, $c \neq 0$, and $d = 2\log_{y^3}(b^3)$, then find the value of $\frac{d}{c}$.

13.3.4 Prove that if $|a| \neq 1$, $|c| \neq 1$, $ac \neq 0$, and we have both $a^x = c^q$ and $c^y = a^z$, then $xy = qz$.

13.3.5 To the nearest thousandth, $\log_{10} 2$ is 0.301 and $\log_{10} 3$ is 0.477. Which of the following is the best approximation of $\log_5 10$?

$$\frac{8}{7}, \quad \frac{9}{7}, \quad \frac{10}{7}, \quad \frac{11}{7}, \quad \frac{12}{7}.$$

(Source: AHSME)

13.3.6★ Prove that $(\log_a b)(\log_c d) = (\log_c b)(\log_a d)$. **Hints:** 249

13.4 Using Logarithm Identities

In this section, we use our logarithm identities to solve more challenging problems. In most of these problems, our main strategy is to use our identities to simplify expressions and equations. In fact, that's the main strategy for solving a great deal of algebra problems:

> **Concept:** Use basic tools to simplify expressions and to convert complicated equations into simpler equations you know how to solve.

Try using this strategy on the following problems.

Problems

Problem 13.24: In this problem, we find all values of x such that $2\log_3(x+4) - \log_3(4x-11) = 2$.

(a) Use logarithm identities to write the left side as a single logarithm.

(b) Convert your equation from part (a) to exponential form, and solve the resulting equation.

(c) Confirm that your solutions satisfy the original equation.