Exercises

13.5.1 Suppose that $\log_{10} xy^3 = 1$ and $\log_{10} x^2y = 1$. What is $\log_{10} xy$? *(Source: AMC 12)*

13.5.2 If $60^a = 3$ and $60^b = 5$, then find $12^{(1-a-b)/[2(1-b)]}$. *(Source: AHSME)*

13.5.3 If $\log_2 p = \log_3 r = \log_{36} 17$, then determine pr. *(Source: Mandelbrot)* **Hints:** 67

13.5.4 Suppose that $a > b > 0$. Solve the equation $(a^4 - 2a^2b^2 + b^4)^{y-1} = (a-b)^{2y}(a+b)^{-2}$ for y in terms of a and b.

13.5.5 Find all x such that $x^{\log_{10} x} = \frac{x^3}{100}$. *(Source: AHSME)*

13.6 Natural Logarithms and Exponential Decay

Problems

Problem 13.33: Let $f(n) = \left(1 + \frac{1}{n}\right)^n$ and $g(n) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$.

(a) Evaluate $f(1)$, $f(2)$, $f(5)$, and $f(10)$. (Yes, you can use a calculator on this problem.)

(b) Evaluate $g(1)$, $g(2)$, $g(5)$, and $g(10)$.

(c) Use a calculator or computer to evaluate $f(n)$ and $g(n)$ for large values of n. Notice anything interesting?

We start this section by introducing a special constant that, much like π, has a great many uses in mathematics and science.

Problem 13.33: Let $f(n) = \left(1 + \frac{1}{n}\right)^n$ and $g(n) = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$. Use a calculator or computer to evaluate $f(n)$ and $g(n)$ for large values of n. Notice anything interesting?

Solution for Problem 13.33: The table below shows $f(n)$ and $g(n)$ to the nearest 10^{-8} for several values of n.

n	$f(n)$	$g(n)$
1	2.00000000	2.00000000
2	2.25000000	2.50000000
5	2.48832000	2.71666668
10	2.59374246	2.71828180
100	2.70481383	2.71828183
1000	2.71692393	2.71828183
100000	2.71826824	2.71828183
10000000	2.71828169	2.71828183

It looks like both $f(n)$ and $g(n)$ approach constant values as n gets very large. Moreover, it looks like