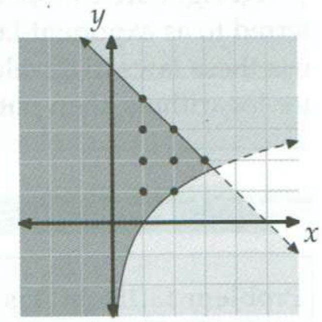
Concept: Not all problems have nice neat solutions. Sometimes a little casework is required to find a solution.

Solution 2: Graphing. We graph the two inequalities as shown at right. The graph of $y \le 5 - x$ consists of the diagonal line shown and the region below this line. The graph of $y \ge \log_2 x$ consists of the graph of $y = \log_2 x$ and the shaded region above this graph. The points that satisfy both these inequalities are the dark shaded region and the solid portions of the graphs of $y = 5 - x$ and $y = \log_2 x$.



We see that there are 8 lattice points (points with integer coordinates) with positive coordinates that are either in the darkest shaded region, or on the solid lines in the diagram. These points are in bold in the diagram. So, there are 8 ordered pairs of positive integers that satisfy both inequalities. □

Our first solution is more rigorous than our second solution. While graphs can be excellent guides to finding an answer, they are not proofs. For example, our graph above of $y = \log_2 x$ appears to pass through $(2, 1)$. But we can't just trust what the graph appears to be in order to be completely confident in our answer. We should confirm that it passes through $(2, 1)$ by testing that $(2, 1)$ satisfies the equation $y = \log_2 x$. Similarly, our graph of $y = \log_2 x$ appears to pass between the points $(3, 1)$ and $(3, 2)$. While this looks obvious, our casework solution is more thorough in explaining why this is the case.

Concept: Graphing can be a useful tool when working with unusual equations or inequalities.

WARNING!! Graphs are excellent for providing intuition about a problem, but they shouldn't be used as a key part of a proof or a rigorous solution to a problem.

Exercises

13.2.1 Evaluate each of the following:

(a) $\log_4 256$ (b) $\log_3 \frac{1}{9}$ (c) $\log_{125} 5$

13.2.2 Evaluate each of the following:

(a) $\log_{125} 25$ (b) $\log_{2\sqrt{2}} 16$ (c) $\log_{1/3} 9$

13.2.3 Solve the equation $3 \cdot 2^x - 9 = 14$.

13.2.4 Solve the equation $\log_2(x^2 - 2x - 7) = 3$.

13.2.5 Find the domain and range of each of the following:

(a) $f(x) = \log_{1/5} x$ (b) $g(x) = \log_6(\log_{1/5} x)$

13.2.6★ Let $f(x) = \log_3 (x - 1)$ and $g(x) = \sqrt{\frac{x}{x-1}}$. Find the domain of $g(f(x))$. **Hints:** 68