**Concept:** If you have an equation with constants raised to powers, it's often helpful to write all those constants with the same base if possible.

(b) Our first step is to note that $27 = 3^3$, so we can write each side as a power of 3. Specifically, on the left we have $3^{27^t} = 3^{(3^3)^t} = 3^{3^{3t}}$ and on the right, we have $27^{3^t} = (3^3)^{3^t} = 3^{3\cdot 3^t} = 3^{3^{t+1}}$, so our equation is

$$3^{3^{3t}} = 3^{3^{t+1}}$$

Because the base of both sides is 3, the exponents of both sides must be equal, which gives us $3^{3t} = 3^{t+1}$. Again, the base of both sides is 3, so the exponents must be equal. This gives us $3t = t + 1$, from which we find $t = 1/2$.

□

**Exercises**

**13.1.1** Find all $t$ such that $16^t = 8^{2-t}$.

**13.1.2** The equation $2^{x^2} = 32^{3x+8}$ has two real solutions. Find their product.

**13.1.3** A function $f$ is **monotonically decreasing** if $f(x) < f(y)$ whenever $x > y$. Let $g(x) = a^x$, where $0 < a < 1$.

(a) Show that $g$ is monotonically decreasing.

(b) Show that if $g(x) = g(y)$, then $x = y$.

**13.1.4** Find the maximum value of $3^{-x^2+2x+3}$.

**13.1.5** Find all values of $y$ such that $4^y - 12 \cdot 2^y = -32$. **Hints:** 93

## 13.2 Introduction to Logarithms

In addition to helping us solve equations, the fact that exponential functions with base greater than 1 are monotonically increasing tells us something very important about them.

**Problems**

**Problem 13.4:** Suppose that $f(x) = a^x$, where $a$ is constant such that $a > 1$. Show that $f(x)$ has an inverse. (If you don't remember what an inverse function is, read Section 2.4, then try this problem again.)

**Problem 13.5:** Let $g(x) = 2^x$. Find $g^{-1}(32)$.

**Problem 13.4:** Suppose that $f(x) = a^x$, where $a$ is constant such that $a > 1$. Show that $f(x)$ has an inverse.