Exercises

3.3.1 Find $|(10 + 24i)(8 - 6i)|$ without finding the product $(10 + 24i)(8 - 6i)$.

3.3.2 Show that $|z| = |\overline{z}|$.

3.3.3 Solve $3z + 4\overline{z} = 12 - 5i$ for z.

3.3.4 Solve each of the following equations:

(a) $z^2 = 2i$ (b) $z^2 = -5 + 12i$ (c) $z^2 = 24 - 10i$

3.3.5 Let $z = 3 + 4i$ and $w = 5 - 12i$. Evaluate the following expressions:

(a) $\left|\frac{1}{z}\right|$ (b) $\frac{1}{|z|}$ (c) $\left|\frac{z}{w}\right|$ (d) $\frac{|z|}{|w|}$

Compare your answers in parts (a) and (b), and compare your answers in parts (c) and (d). Notice anything interesting? (You should!) Prove that your observations will hold for any nonzero complex numbers.

3.3.6 Find all complex numbers z such that $z/\overline{z}$ is

(a) a real number. (b) an imaginary number.

3.3.7★ Two solutions of $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$ are pure imaginary numbers. Find these two solutions. *(Source: ARML)* **Hints:** 213

3.4 Graphing in the Complex Plane

We graph an equation in terms of x and y in the Cartesian plane by plotting all the points (x, y) that satisfy the equation. Similarly, we graph an equation in terms of a complex number z in the complex plane by plotting all the values of z that satisfy the equation.

Problems

Problem 3.19: Show that $\text{Re}(z) = \frac{z+\overline{z}}{2}$ and $\text{Im}(z) = \frac{z-\overline{z}}{2i}$.

Problem 3.20: Graph in the complex plane all complex numbers z that satisfy each of the following:

(a) $\frac{z+\overline{z}}{2} = 6$

(b) $z - \overline{z} = -2\sqrt{7}i$

(c) $(3 + 2i)z + (3 - 2i)\overline{z} = 36$

Problem 3.21: Let w be a constant complex number and c be a nonnegative real constant. Describe the graphs of each of the following:

(a) $|z| = 3$

(b) $|z - 4| = 3$

(c) $|z + 5 - 4i| = 2\sqrt{2}$

(d) $|z - w| = c$