3.2.4 Find the distance between the points $4 + 7i$ and $-3 - 17i$ in the complex plane.

3.2.5 Show that the midpoint of the segment connecting z_1 and z_2 on the complex plane is $(z_1 + z_2)/2$.

3.2.6

(a) Find the magnitude of $\dfrac{1+2i}{2+i}$.

(b) Find the magnitude of $\dfrac{6+11i}{11+6i}$. (You can use a calculator for this part.)

(c) Notice anything interesting? Can you generalize your observations from the first two parts?

3.2.7★ Four complex numbers lie at the vertices of a square in the complex plane. Three of the numbers are $1 + 2i$, $-2 + i$ and $-1 - 2i$. What is the fourth number? *(Source: AMC 12)*

3.3 Real and Imaginary Parts

Problems

Problem 3.13: Let z and w be complex numbers.

(a) Let $z = a + bi$ and $w = c + di$, where a, b, c, and d are real. Show that $\overline{z+w} = \overline{z} + \overline{w}$.

(b) Show that $\overline{zw} = \overline{z} \cdot \overline{w}$.

Problem 3.14:

(a) Show that $\overline{\overline{z}} = z$ for all complex numbers z.

(b) Show that $\overline{z} = z$ if and only if z is real.

(c) Show that $\overline{z} = -z$ if and only if z is imaginary.

Problem 3.15:

(a) Prove that $z\overline{z} = |z|^2$ for all complex numbers z.

(b) Prove that $|zw| = |z||w|$ for all complex numbers z and w.

Problem 3.16: Solve the equation $z + 2\overline{z} = 6 - 4i$ for z.

Problem 3.17: In this problem, we find all complex numbers z such that $z^2 = 21 - 20i$.

(a) Let $z = a + bi$ in the given equation. Find a system of equations involving a and b.

(b) Solve for b in terms of a in one of the equations, and substitute the expression you found for b into the other equation.

(c) Solve the equation you formed in part (b) for all possible values of a. **Hints:** 90

(d) Find all complex numbers z such that $z^2 = 21 - 20i$.