Problem 3.12:

(a) Evaluate $|5 - 12i|$ and $|3 - 3i|$.

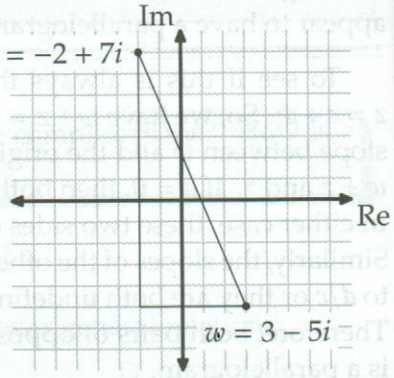
(b) Suppose $w = 3 - 5i$ and $z = -2 + 7i$. Find $|w - z|$. Find the distance between w and z on the complex plane. Notice anything interesting?

(c) For any complex numbers w and z, how is $|w - z|$ related to the distance between w and z on the complex plane?

Solution for Problem 3.12:

(a) We have $|5 - 12i| = \sqrt{5^2 + (-12)^2} = 13$ and $|3 - 3i| = \sqrt{3^2 + (-3)^2} = 3\sqrt{2}$.

(b) We have $w - z = 5 - 12i$, and in the previous part we found that $|5 - 12i| = 13$. We plot the two points in the diagram at right. We see that the horizontal distance between them is 5 and the vertical distance between them is 12, so the segment connecting the two points is the hypotenuse of a right triangle with legs of lengths 5 and 12. Therefore, the Pythagorean Theorem tells us that the distance between w and z on the complex plane is $\sqrt{5^2 + 12^2} = 13$.



Notice that the distance between w and z equals $|w - z|$. Is this a coincidence?

(c) No, it's not a coincidence. Suppose $w = a + bi$ and $z = c + di$. Finding the distance between w and z in the complex plane is the same as finding the distance between (a, b) and (c, d) on the Cartesian plane. So, the distance between w and z is $\sqrt{(a - c)^2 + (b - d)^2}$.

Because $w - z = (a + bi) - (c + di) = (a - c) + (b - d)i$, we have

$$|w - z| = \sqrt{(a - c)^2 + (b - d)^2}.$$

This tells us that:

Important: The distance between complex numbers w and z on the complex plane is $|w - z|$.

□

Exercises

3.2.1 Plot each of the following in the complex plane:

(a) $4 + 7i$ (b) $-6 - 2i$ (c) $(3 + i)(-2 + 5i)$

3.2.2 Find the magnitude of each of the following complex numbers:

(a) $24 - 7i$ (b) $2 + 2\sqrt{3}i$ (c) $(1 + 2i)(2 + i)$

3.2.3 Let $w = 3 + 5i$ and $z = 12 + 2i$. Find the area of the convex quadrilateral in the complex plane that has vertices w, z, $\overline{w}$, and $\overline{z}$.