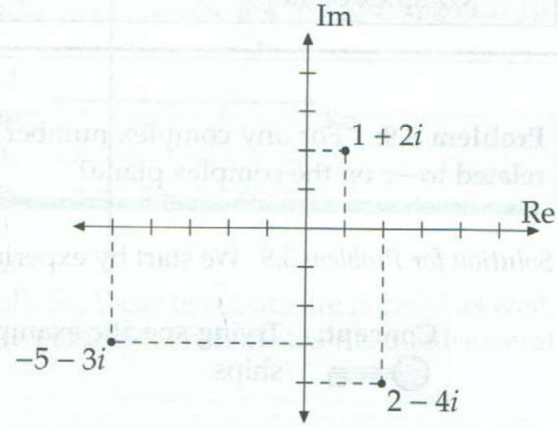
3.1.6 Solve each of the following equations for z:

(a) $\dfrac{z+3i}{z-3} = 2$ (b) $\dfrac{1+2i}{3z} = 4+5i$ (c) $3z + \dfrac{z}{1+i} = 10-4i$

3.1.7 Let $S = i^n + i^{-n}$, where n is an integer. Find the total number of possible distinct values of S. *(Source: AHSME)*

3.2 The Complex Plane

A complex number can be represented as a point in the **complex plane**. Like the coordinate plane, the complex plane has two axes, a horizontal **real axis** for the real part, and a vertical **imaginary axis** for the imaginary part. So, plotting the complex number $x + yi$ on the complex plane is the same as plotting the point (x, y) in the Cartesian coordinate plane. The figure at right shows the numbers $1 + 2i$, $-5 - 3i$, and $2 - 4i$ plotted in the complex plane.



Just as the point $(0, 0)$ is called the origin on the Cartesian plane, the point that represents 0 on the complex plane is also called the **origin** of the complex plane.

The real axis of the complex plane is labeled Re and the imaginary axis is labeled Im. Similarly, we can use $\text{Re}(z)$ to refer to the real part of the complex number z, and use $\text{Im}(z)$ to refer to the imaginary part of z. So, for example $\text{Re}(2 - 4i) = 2$ and $\text{Im}(2 - 4i) = -4$. Some sources use $\Re(z)$ and $\Im(z)$ in place of $\text{Re}(z)$ and $\text{Im}(z)$.

Problems

Problem 3.9:

(a) Let $w = 2 + 3i$. Plot w, $\overline{w}$, and $-w$ on the complex plane.

(b) For any complex number z, how is z related to $\overline{z}$ on the complex plane? How is z related to $-z$ on the complex plane?

Problem 3.10:

(a) Let $w = 2 + 4i$ and $z = 3 - 2i$. Plot w, z, and $w + z$ on the complex plane. Connect the origin to w, then connect w to $w + z$, then connect $w + z$ to z, then connect z to the origin. Notice anything interesting?

(b) Suppose w and z are nonzero complex numbers such that w/z is not a real constant. (In other words, there is no real number a such that $w = az$.) Connect the origin to w, then connect w to $w + z$, then connect $w + z$ to z, then connect z to the origin. What type of quadrilateral must result? Why?