Notice that the answer to part (b) is the conjugate of the answer to part (a), and the answer to part (d) is the conjugate of the answer to part (c). This isn't a coincidence. We'll explore why in Section 3.3. See if you can figure out why on your own before getting there.

Problem 3.8: Find z such that $\dfrac{2z-3i}{z+4} = -5+i$.

Solution for Problem 3.8: We begin by multiplying both sides by the denominator of the left side, which gives us $2z - 3i = (-5+i)(z+4)$. Expanding the right side then gives $2z - 3i = -5z - 20 + iz + 4i$. Putting all terms with z on one side and terms without z on the other gives us

$$7z - iz = -20 + 7i.$$

Factoring z out of the left side gives $z(7-i) = -20+7i$. Dividing both sides by $7-i$ then gives us

$$z = \frac{-20+7i}{7-i} = \frac{-20+7i}{7-i}\cdot\frac{7+i}{7+i} = \frac{-147+29i}{50} = -\frac{147}{50} + \frac{29}{50}i.$$

□

Sidenote: Mathematicians frequently use w and z as variables to represent complex numbers, and use x and y as variables to represent real numbers. Of course, this is not a hard-and-fast rule! As you'll see in this book, the variable x is so universally common that we will have many equations in terms of x to which some solutions turn out to be nonreal.

Exercises

3.1.1 Find all z such that $4z^2 + 12 = 0$.

3.1.2 Let $a = 3+4i$ and $b = 12-5i$. Compute each of the following:

(a) $a - b$

(b) ab

(c) $a^2 + 3a + 2$

(d) $\dfrac{a}{b} + \dfrac{\overline{a}}{b}$

3.1.3 Write each of the following as a complex number:

(a) $\overline{2i + 7 - 2i}$

(b) $\dfrac{1}{\overline{2+3i} + \overline{1+2i}}$

3.1.4 $f(x) = \dfrac{x^6+x^4}{x+1}$. Find each of the following:

(a) $f(i)$

(b) $f(-i)$

(c) $f(i-1)$

3.1.5 What is $(i - i^{-1})^{-1}$? *(Source: AHSME)*