Chapter 4 Vectors in Two Dimensions

Basic Practice

1. $\mathbf{a} = \mathbf{h}$; $\mathbf{b} = \mathbf{k}$; $\mathbf{c} = \mathbf{g}$; $\mathbf{d} = \mathbf{j}$; $\mathbf{e} = \mathbf{f}$

2. (a) $|\overrightarrow{AB}| = 2$

$\overrightarrow{AB}$ is along the negative direction of the y-axis.

(b) $|\overrightarrow{CD}| = 1\frac{1}{2}$

$\overrightarrow{CD}$ is along the negative direction of the x-axis.

(c) $|\overrightarrow{EF}| = \sqrt{5}$

$\overrightarrow{EF}$ is upwards along a line with gradient $= -2$.

(d) $|\overrightarrow{GH}| = \sqrt{2}$

$\overrightarrow{GH}$ is downwards along a line with gradient $= -1$.

(e) $|\overrightarrow{KL}| = \sqrt{2^2 + \left(\frac{1}{2}\right)^2} \approx 2.06$

$\overrightarrow{KL}$ is upwards along a line with gradient $= \frac{1}{4}$.

(f) $|\overrightarrow{MN}| = \sqrt{20}$

$\overrightarrow{MN}$ is downwards along a line with gradient $= -\frac{1}{2}$.

3. (a) $5\frac{1}{2}\mathbf{p}$ (b) $2\mathbf{p} + 2\mathbf{q}$ (c) $\mathbf{p} + 2\mathbf{q}$

(d) $1\frac{1}{2}\mathbf{p} - \mathbf{q}$ (e) $\mathbf{p} - 3\mathbf{q}$ (f) $-1\frac{1}{2}\mathbf{p} + 3\mathbf{q}$

(g) $-2\frac{1}{2}\mathbf{p} + 3\mathbf{q}$ (h) ~~$2\mathbf{p} + 2\mathbf{q}$~~ $-2\mathbf{p} - 2\mathbf{q}$

4. (a) $\overrightarrow{SR}$ (b) $\overrightarrow{SP}$ (c) $\overrightarrow{RP}$ (d) O
(e) $\overrightarrow{QP}$ (f) $\overrightarrow{QR}$

5. (a) $2\mathbf{p}$ (b) $-\mathbf{q}$ (c) $\mathbf{p} + \mathbf{q}$ (d) $-\mathbf{p} - \mathbf{q}$
(e) $-\mathbf{p} + \mathbf{q}$ (f) $-2\mathbf{p} - \mathbf{q}$

6. (a) $\frac{1}{2}\mathbf{p}$ (b) $\frac{1}{3}\mathbf{r}$ (c) $\mathbf{r} + \frac{1}{2}\mathbf{p}$

(d) $\mathbf{p} + \frac{1}{3}\mathbf{r}$ (e) $-\frac{1}{2}\mathbf{p} + \frac{2}{3}\mathbf{r}$ (f) $\mathbf{p} - \frac{2}{3}\mathbf{r}$

8. (a) (i) $\begin{pmatrix} 6 \\ -15 \end{pmatrix}$ (ii) $\begin{pmatrix} 6 \\ -14 \end{pmatrix}$ (iii) $\begin{pmatrix} 12 \\ -29 \end{pmatrix}$

(iv) $\begin{pmatrix} -5 \\ 12 \end{pmatrix}$

(b) $3|\overrightarrow{OP}| \approx 16.2$, $|-2\overrightarrow{OQ}| \approx 15.2$,

$|3\overrightarrow{OP} - 2\overrightarrow{OQ}| \approx 31.4$, $|\overrightarrow{PQ}| \approx 13$

9. (a) Coordinates of B is (4, 6)
Coordinates of C is (5, 1)

(b) (i) $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ (ii) 5.10

10. (a) (i) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (ii) $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \mathbf{c}$

(iii) $\frac{1}{2}\left(\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} - \mathbf{c}\right)$ (iv) $\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{c}$

(v) $\frac{1}{2}(\mathbf{b} - \mathbf{c})$

(b) $\overrightarrow{CB} = \mathbf{b} - \mathbf{c} = 2\overrightarrow{AD}$
$\therefore AD \parallel CB$

11. (a) (i) $3\mathbf{p}$ (ii) $-2\mathbf{p}$ (iii) $-3\mathbf{q}$
(iv) $-2(\mathbf{p} + \mathbf{q})$ (v) $-3(\mathbf{p} + \mathbf{q})$
(b) (ii) 2 : 3

Further Practice

12. (a) (i) $\overrightarrow{LF}$ and $\overrightarrow{CE}$ (ii) $\overrightarrow{EG}$ and $\overrightarrow{FH}$
(b) (i) 4 cm (ii) 16 cm
(c) No. This is because $|\overrightarrow{LF} + \overrightarrow{FH} + \overrightarrow{HJ} + \overrightarrow{JL}| = 0$ cm

13. (a) 17 (b) −30
(c) (i) $\begin{pmatrix} 31 \\ 38 \end{pmatrix}$ (ii) 49.0

15. (a) $\overrightarrow{BD}$ (b) $\overrightarrow{ED}$ (c) $\overrightarrow{BC}$

16. (a) (i) 13.4 (ii) 6
(b) $\triangle AOC$ is isosceles. $\angle AOC = 90°$.

17. (a) $|\overrightarrow{OP}| \approx 6.32$, $|\overrightarrow{OQ}| \approx 8.06$
(b) 25.6°

18. (b) $s = \frac{1}{4}$, $t = 1\frac{1}{2}$
(c) (ii) $\overrightarrow{OR} = \frac{1}{2}\mathbf{a}$, $\overrightarrow{BR} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

19. (a) 3
(b) $\overrightarrow{AC} = \begin{pmatrix} 8 \\ 16 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$
(c) (i) $\begin{pmatrix} 4 \\ -12 \end{pmatrix}$ (ii) (4, −12)
(d) $\begin{pmatrix} 12 \\ 4 \end{pmatrix}$

20. (a) (i) $\frac{2}{3}(\mathbf{a} + \mathbf{c})$ (ii) $\mathbf{a} + \mathbf{c}$
(b) $\overrightarrow{OX} = \frac{2}{3}\overrightarrow{OY}$ and O are common.
$\therefore O$, X and Y lie on the same line.

21. (a) (i) $4\mathbf{p} - \mathbf{q}$ (ii) $\frac{1}{3}(14\mathbf{p} + \mathbf{q})$
(iii) $\frac{2}{9}(14\mathbf{p} + \mathbf{q})$ (iv) $\frac{7}{9}(4\mathbf{p} - \mathbf{q})$
(b) $\overrightarrow{QT} = \frac{7}{9}\overrightarrow{QS}$
Q, S and T lie on the same line.
(c) (i) 7 : 9 (ii) 4 : 7 (iii) 4 : 9