Math 9 Fall 2015 Final Exam

1. $\frac{1}{2}W + 3\begin{bmatrix} -1 & 4 \\ 8 & -3 \end{bmatrix} = 4\begin{bmatrix} 0 & -2 \\ 3 & 8 \end{bmatrix}$

$$\frac{1}{2}W = \begin{bmatrix} 0 & -8 \\ 12 & 32 \end{bmatrix} - \begin{bmatrix} -3 & 12 \\ 24 & -9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -20 \\ -12 & 41 \end{bmatrix}$$

$$W = \begin{bmatrix} 6 & -40 \\ -24 & 82 \end{bmatrix}$$

3. $h\begin{bmatrix} 2 \\ 6 \end{bmatrix} + k\begin{bmatrix} 3 \\ 9 \end{bmatrix} = \begin{bmatrix} 16 \\ 48 \end{bmatrix}$

$$\begin{bmatrix} 2h \\ 6h \end{bmatrix} + \begin{bmatrix} 3k \\ 9k \end{bmatrix} = \begin{bmatrix} 16 \\ 48 \end{bmatrix}$$

$\begin{cases} 2h + 3k = 16 \\ 6h + 9k = 48 \end{cases}$ two ~~gues~~ equations one the same,

so there are many h and k that satisfy $2h + 3k = 16$

2. $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$A + 2A + 3A + \cdots + nA$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix} + \cdots + \begin{bmatrix} n & 0 \\ 0 & 3n \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+\cdots+n & 0 \\ 0 & 3(1+2+\cdots+n) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{n(n+1)}{2} & 0 \\ 0 & \frac{3n(n+1)}{2} \end{bmatrix}$$

4. $\begin{bmatrix} -2 & \sin\theta \\ -5 & 9 \\ 3x & -7 \end{bmatrix} = \begin{bmatrix} -2 & 0.5 \\ -5 & 9 \\ x+4 & -7 \end{bmatrix}$

$\sin\theta = 0.5 \quad \theta = 30^\circ, 150^\circ$

$3x = x + 4 \quad 2x = 4 \quad x = 2$

5. $\begin{bmatrix} 2 & 3 \\ 7 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 37 \end{bmatrix}$

$\begin{cases} 2x + 3y = 4 \\ 7x - y = 37 \end{cases} \quad y = 7x - 37$

$2x + 3(7x - 37) = 4$

$2x + 21x - 111 = 4 \quad 23x = 115$

$x = 5 \quad y = 35 - 37 = -2$

6. $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A \cdot B = I \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{cases} ap + br = 1 \\ cp + dr = 0 \end{cases}$ $\quad \begin{cases} aq + bs = 0 \\ cq + ds = 1 \end{cases}$

$p = -\frac{dr}{c}$ $\quad q = -\frac{bs}{a}$

$-\frac{adr}{c} + br = 1$ $\quad -\frac{cbs}{a} + ds = 1$

$-adr + bcr = c$ $\quad -cbs + ads = a$

$r(bc - ad) = c$ $\quad s(ad - cb) = a$

$r = \frac{c}{bc - ad}$ $\quad s = \frac{a}{ad - cb}$

$p = \frac{-d}{bc - ad}$ $\quad q = \frac{-b}{ad - cb}$

We can further verify that

$B \cdot A = I$ too

7. $\frac{x-9}{3} = 5 + \frac{17}{x+2}$

$\frac{x-9}{3} = \frac{5x+27}{x+2}$

$(x-9)(x+2) = 15x + 81$

$x^2 - 7x - 18 = 15x + 81$

$x^2 - 22x - 99 = 0$

$x = \frac{22 \pm \sqrt{22^2 + 4 \cdot 99}}{2} = \frac{22 \pm 29.66}{2}$

$x_1 = \frac{22 + 29.66}{2} = \frac{51.66}{2} = 25.83$ $\quad x_2 = \frac{22 - 29.66}{2} = \frac{-7.66}{2} = -3.83$

8. $\frac{x}{x+4} - \frac{4x+5}{x+2} = 0$

$\frac{x}{x+4} = \frac{4x+5}{x+2}$

$x^2 + 2x = 4x^2 + 21x + 20$

$3x^2 + 19x + 20 = 0$

$(x+5)(3x+4) = 0$

$x = -5, \quad x = -\frac{4}{3}$

9. $5t - 8 < 16 + 3t$

$2t < 24 \quad t < 12$

10. $4x - 5 > 7x + 10$

$3x < -15 \quad x < -5$

$x \geq -8$, all integers are:

$-8, -7, -6$

11. A = { letter from 'TOOTH' }

a. = { T, O, H }

b. $n(A) = 3$

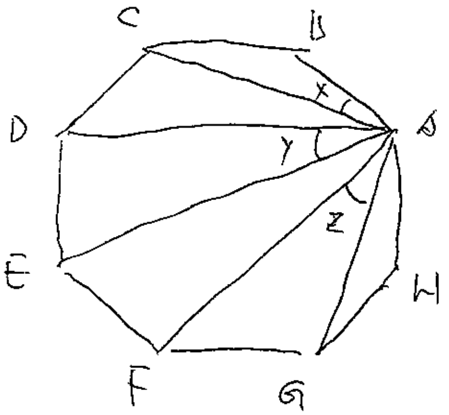
c. { }, {T}, {O}, {H}, {T,O}, {T,H}, {O,H}, {T, O, H}

d. $B = \{\text{letter from 'SOOTH'}\}$

$= \{S, O, T, H\}$

$A \subset B$, A is a subset of B.

12.



The sum of all interior angles is $(n-2)\cdot 180 = (8-2)\cdot 180 = 6\cdot 180$

$= 1080$

Because it is a regular octagon. every interior angle is the same.

So $\angle BAH = \frac{1080}{8} = 135°$

$\triangle CBA$, $CB = BA$, so

$\angle BAC = \angle BCA$

$2X + 135 = 180 \quad X = \frac{45}{2} = 22.5°$

In $AFGH$, sum of all interior angles are $(4-2)\cdot 180 = 360$

$\therefore \angle FAH + \angle AHG = 180$

$\angle FAH = 180 - 135° = 45°$

Since $\angle GAH = X = 22.5°$

$\therefore z = \angle FAG = 45° - 22.5° = 22.5°$

$\angle DAF = \angle BAH - 2X - 2Z$

$= 135° - 45° - 45° = 45°$

Since $\triangle ADE \cong \triangle AFE$

$\therefore \angle DAE = \angle EAF$

$\therefore y = \angle DAE = \frac{\angle DAF}{2} = \frac{45°}{2} = 22.5°$

13. a. $\because \triangle AEF$ similar to $\triangle ACD$

$\frac{AF}{AD} = \frac{AE}{AC} \qquad \frac{6}{6+4} = \frac{9}{9+X}$

$\frac{6}{10} = \frac{9}{X+9} = \frac{3}{5}$

$3X+27 = 45 \quad 3X = 18 \quad X = 6$

$\frac{AF}{AD} = \frac{EF}{CD} \qquad \frac{6}{10} = \frac{y}{10} \cancel{=10}$

$\cancel{y=10} \quad y = 6$

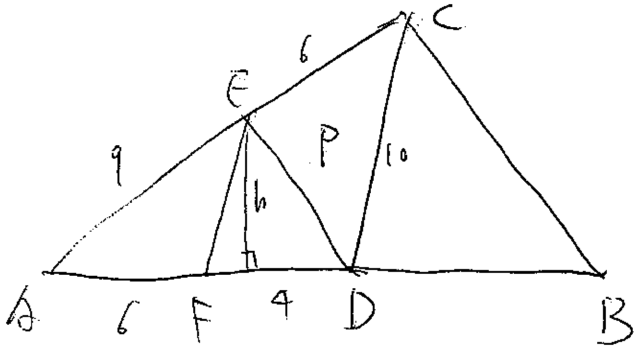
$\because \triangle AED$ similar to $\triangle ACB$

$\frac{AE}{AC} = \frac{AD}{AB} \qquad \frac{9}{9+6} = \frac{10}{10+Z}$

$\cancel{9+6} \qquad \frac{10}{10+Z} = \frac{9}{15} = \frac{3}{5}$

$3Z + 30 = 50 \qquad Z = \frac{20}{3}$

b. ~~30 = Area of △ADE = 1/2~~



$30 = \text{Area of } \triangle AED = \frac{1}{2}(6+4)\cdot h$

$= \frac{1}{2}\cdot 10\cdot h \qquad h = 6$

$A_{\triangle AEF} = \frac{1}{2}\cdot 6\cdot h = \frac{1}{2}\cdot 6\cdot 6 = 18\ cm^2$

$\triangle AEF$ similar to $\triangle ACD$

$\frac{A_{\triangle AEF}}{A_{\triangle ACD}} = \frac{6^2}{10^2} \qquad \frac{18}{30+p} = \frac{36}{100} = \frac{9}{25}$

$9p + 270 = 25\cdot 18 \qquad p = 20\ cm^2$

$A_{BCED} = r$

$\triangle AED$ similar to $\triangle ACB$

$\frac{A_{\triangle AED}}{A_{\triangle ACB}} = \frac{9^2}{15^2} \qquad \frac{30}{30+r} = \frac{81}{225} = \frac{9}{25}$

$9r + 9\cdot 30 = 30\cdot 25$

$r = \frac{160}{3} = 53\frac{1}{3}\ cm^2$

Bonus

1. $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

2. $A \times B = D \qquad m\times p$

$d_{ij} = \sum_{k=1}^{n} a_{ik}\cdot b_{kj} \qquad i = 1,\dots m \quad j = 1,\dots p$

$D \times C = E \qquad m\times q$

$e_{ij} = \sum_{l=1}^{p} d_{il}\cdot c_{lj} \qquad i = 1,\dots m \quad j = 1,\dots q$

$= \sum_{l=1}^{p} \sum_{k=1}^{n} a_{ik}\cdot b_{kl}\cdot c_{lj}$

$B \times C = F \qquad n\times q$

$f_{ij} = \sum_{l=1}^{p} b_{il}\cdot c_{lj}$

$A \times F = G \qquad m\times q$

$g_{ij} = \sum_{k=1}^{n} a_{ik}\cdot f_{kj}$

$= \sum_{k=1}^{n} a_{ik}\cdot \sum_{l=1}^{p} b_{kl}\cdot c_{lj}$

$= \sum_{l=1}^{p} \sum_{k=1}^{n} a_{ik}\cdot b_{kl}\cdot c_{lj} = e_{ij}$